10th Class 2018

Math (Science)	Group-I	PAPER-II
Time: 2.10 Hours	(Subjective Type)	Max. Marks: 60

(Part-I)

2. Write short answers to any SIX (6) questions: (12)

(i) Solve by factorization: $x^2 - x - 20 = 0$

Ans Given,

$$x^2 - x - 20 = 0$$

By factorization method,

$$x^2 - 5x + 4x - 20 = 0$$
$$x(x-5) + 4(x-5) = 0$$
$$(x+4)(x-5) = 0$$

For the both values of 'x':

Firstly,

$$x + 4 = 0$$
$$x = -4$$

and

$$x - 5 = 0$$
$$x = 5$$

So, $\{-4, 5\}$ Ans

(ii) Define radical equation.

Ans An equation involving expression under the radical sign is called a radical equation.

(iii) Find the discriminant of the following equation:

$$6x^2 - 8x + 3 = 0$$

Ans

$$6x^2 - 8x + 3 = 0$$

Here, $a = 6$, $b = -8$, $c = 3$

Discriminant $= b^2 - 4ac$

$$= (-8)^2 - 4(6)(3)$$
$$= 64 - 72$$
$$= -8$$

(iv) Evaluate: $(1 - \omega - \omega^2)^7$

Ans Given, $(1 - \omega - \omega^2)^7$

$= [1 - (\omega + \omega^2)]^7$ (i)

Using $1 + \omega + \omega^2 = 0$

$\omega + \omega^2 = -1$

By putting in (i),

$= [1 - (-1)]^7$

$= [1 + 1]^7$

$= 2^7$

$= 128$

(v) Without solving, find the sum and the product of the roots of quadratic equation: $x^2 - 5x + 3 = 0$.

Ans Here, $a = 1, b = -5, c = 3$

Sum of the roots,

$$S = \alpha + \beta = \frac{-b}{a}$$

$$= \frac{-(-5)}{1}$$

$$= \frac{5}{1}$$

$$S = 5$$

Product of the roots,

$$P = \alpha\beta = \frac{c}{a}$$

$$= \frac{3}{1}$$

$$P = 3$$

(vi) Use synthetic division to find the quotient and the remainder when: $(4x^3 - 5x + 15) \div (x + 3)$.

Ans $P(x) = 4x^3 - 5x + 15$

$= 4x^3 + 0x^2 - 5x + 15$

Here, $x - a = x + 3$

$-a = 3$

$a = -3$

Using synthetic division,

	4	0	−5	15
−3	↓	−12	36	−93
	4	−12	31	−78

$$Q(x) = 4x^2 - 12x + 31$$

Remainder (R) = −78

(vii) Find the value of p, if the ratios 2p + 5 : 3p + 4 and 3 : 4 are equal.

Ans Given condition,

$$2p + 5 : 3p + 4 = 3 : 4$$

$$\Rightarrow \quad \frac{2p+5}{3p+4} = \frac{3}{4}$$

$$4(2p + 5) = 3(3p + 4)$$

$$8p + 20 = 9p + 12$$

$$8p - 9p = 12 - 20$$

$$-p = -8$$

$$p = 8$$

(viii) Define joint variation.

Ans A combination of direct and inverse variations of one or more than one variable forms joint variation.

(ix) Find a third proportional to: $a^2 - b^2$, a − b.

Ans Let x = third proportional

$$a^2 - b^2 : (a - b) :: (a - b) : x$$

Product of Extremes = Product of Means

$$x(a^2 - b^2) = (a - b)(a - b)$$

$$x = \frac{(a-b)(a-b)}{(a^2 - b^2)}$$

$$x = \frac{(a-b)(a-b)}{(a+b)(a-b)}$$

$$x = \frac{a-b}{a+b}$$

Thus, the third proportional is $\frac{a-b}{a+b}$.

3. Write short answers to any SIX (6) questions: 12

(i) Define improper fraction.

Ans A rational fraction $\frac{N(x)}{D(x)}$, with $D(x) \neq 0$ is called an improper fraction, if degree of the polynomial N(x) is greater or equal to the degree of the polynomial D(x).

For example:

$$\frac{5x}{x+2}, \frac{3x^2+2}{x^2+7x+12}, \frac{6x^4}{x^3+1}.$$

(ii) Define rational fraction.

Ans An expression of the form $\frac{N(x)}{D(x)}$, where N(x) and D(x) are polynomials in x with real coefficients, is called a rational fraction. The polynomial $D(x) \neq 0$ in the expression.

For example:

$$\frac{2x}{(x-1)(x+2)} \text{ and } \frac{x^2+3}{(x+1)^2(x+2)}.$$

(iii) If X = {1, 4, 7, 9}, Y = {2, 4, 5, 9}, then find X ∪ Y.

Ans

$$X \cup Y = \{1, 4, 7, 9\} \cup \{2, 4, 5, 9\}$$
$$= \{1, 2, 4, 5, 7, 9\}$$

(iv) If A = {a, b}, B = {c, d}, then find A × B and B × A.

Ans

$$A = \{a, b\}$$
$$B = \{c, d\}$$
$$A \times B = \{a, b\} \times \{c, d\}$$
$$= \{(a, c), (a, d), (b, c), (b, d)\}$$
$$B \times A = \{c, d\} \times \{a, b\}$$
$$= \{(c, a), (c, b), (d, a), (d, b)\}$$

(v) Define domain set of relation.

Ans Domain set of relation denoted by Dom R is the set consisting of all the first elements of each ordered pair in the relation.

(vi) Find a and b if (a – 4, b – 2) = (2, 1).

Ans Given, $(a-4, b-2) = (2, 1)$

By comparing both sides, we get

$$a - 4 = 2$$
$$a = 2 + 4$$
$$\boxed{a = 6}$$

and $$b - 2 = 1$$
$$b = 1 + 2$$
$$\boxed{b = 3}$$

(vii) Define arithmetic mean.

Ans Arithmetic mean (or simply called mean) is a measure that determines a value (observation) of the variable under study by dividing the sum of all values (observations) of the variable by their number of observations. We denote arithmetic mean by $\bar{X}$. In symbols, we define

$$\text{Arithmetic Mean} = \bar{X} = \frac{\Sigma X}{n}.$$

(viii) Find arithmetic mean: 12, 14, 17, 20, 24, 29, 35, 45.

Ans The arithmetic mean is

$$\bar{X} = \frac{\Sigma x}{n}$$

$$\bar{X} = \frac{12 + 14 + 17 + 20 + 24 + 29 + 35 + 45}{8}$$

$$= \frac{196}{8}$$

$$\boxed{\bar{X} = 24.5}$$

(ix) The salaries of five teachers in rupees are as: 11,500, 12,400, 15,000, 14,500, 14,800, find range.

Ans The maximum value:

$$X_m = 15{,}000$$

The minimum value:

$$X_o = 11{,}500$$

$$\text{Range} = X_m - X_o$$

$= 15,000 - 11,500$
$= 3,500$

4. Write short answers to any SIX (6) questions: (12)

(i) Define degree.

Ans If we divide the circumference of a circle into 360 equal arcs, then the angle subtended at the centre of the circle by one arc is called one degree, and is denoted by 1°.

(ii) Convert 25°30′ to decimal degree.

Ans
$$25^\circ 30' = 25^\circ + \left(\frac{30}{60}\right)^\circ$$
$$= 25^\circ + 0.5^\circ$$
$$= 25.5^\circ$$

(iii) Find 'l', when $\theta = 180^\circ$, $r = 4.9$ cm.

Ans
$$\theta = 180^\circ \times \frac{\pi}{180^\circ}$$
$$\theta = \pi \text{ radians}$$

As we know that,
$$l = r\theta$$
$$= \pi(4.9)$$
$$= \frac{22}{7}\left(\frac{49}{10}\right)$$
$$= \frac{154}{10} = \frac{77}{5}$$
$$l = 15.4 \text{ cm}$$

(iv) Define obtuse angle.

Ans An angle which is greater than 90° is called obtuse angle.

(v) Define circular area.

Ans The area bounded by the circumference of the circle is called circular area πr^2 is the circular area of a circle of radius r.

(vi) Define length of tangent.

Ans The length of a tangent to a circle is measured from the given point to the point of contact.

(vii) **Define an arc of the circle.**

Ans An arc of a circle is any portion of its circumference.

(viii) **What is meant by sector of a circle?**

Ans A sector of a circle is the plane figure bounded by two radii and the arc intercepted between them.

(ix) **Define circum circle.**

Ans The circle passing through the vertices of triangle ABC is known as circum circle, it radius as circum radius and centre as circum centre.

(Part-II)

NOTE: Attempt THREE (3) questions in all. But question No. 9 is Compulsory.

Q.5.(a) Solve the equation: (4)

$$x^4 - 13x^2 + 36 = 0$$

Ans

$$x^4 - 13x^2 + 36 = 0$$
$$x^4 - 9x^2 - 4x^2 + 36 = 0$$
$$x^2(x^2 - 9) - 4(x^2 9) = 0$$
$$(x^2 - 9)(x^2 - 4) = 0$$
$$x^2 - 9 = 0 \quad ; \quad x^2 - 4 = 0$$
$$x^2 = 9 \quad ; \quad x^2 = 4$$
$$x = \pm 3 \quad ; \quad x = \pm 2$$
$$S.S = \{\pm 3, \pm 2\}$$

(b) Prove that: $x^3 + y^3 = (x + y)(x + \omega y)(x + \omega^2 y)$. (4)

Ans

$$R.H.S = (x + y)(x + \omega y)(x + \omega^2 y)$$
$$= (x + y)[x^2 + \omega^2 xy + \omega xy + \omega^3 y^2]$$
$$= (x + y)[x^2 + (\omega^2 + \omega)xy + \omega^3 y^2]$$
$$= (x + y)[x^2 + (-1)xy + (1)y^2]$$
$$= (x + y)(x^2 - xy + y^2)$$
$$= x^3 + y^3$$
$$= L.H.S$$

Q.6.(a) Find value of $\frac{x+2y}{x-2y} + \frac{x+2z}{x-2z}$ by using theorem of componendo-dividendo, if $x = \frac{4yz}{y+z}$. (4)

Ans Given, $x = \frac{4yz}{y+z}$

$$x = \frac{2y(2z)}{y+z}$$

$$\frac{x}{2y} = \frac{2z}{y+z}$$

By using theorem of componendo-dividendo,

$$\frac{x+2y}{x-2y} = \frac{2z+y+z}{2z-y-z}$$

$$\frac{x+2y}{x-2y} = \frac{3z+y}{z-y} \quad \text{(ii)}$$

Now, $x = \frac{4zy}{y+z}$

$$x = \frac{2z(2y)}{y+z}$$

$$\frac{x}{2z} = \frac{2y}{y+z}$$

By using theorem of componendo-dividendo,

$$\frac{x+2z}{x-2z} = \frac{2y+y+z}{2y-y-z}$$

$$\frac{x+2z}{x-2y} = \frac{3y+z}{y-z} \quad \text{(ii)}$$

By adding (i) and (ii),

$$\frac{x+2y}{x-2y} + \frac{x+2z}{x-2y} = \frac{3z+y}{z-y} + \frac{3y+z}{y-z}$$

$$= \frac{3z+y}{z-y} - \frac{3y+z}{z-y}$$

$$= \frac{3z+y-3y-z}{z-y} = \frac{2z-2y}{z-y}$$

$$= \frac{2(z - y)}{(z - y)}$$
$$= 2$$

(b) Resolve into partial fractions: (4)

$$\frac{9}{(x - 1)(x + 2)^2}$$

Ans Let,

$$\frac{9}{(x + 2)^2(x - 1)} = \frac{A}{x + 2} + \frac{B}{(x + 2)^2} + \frac{C}{x - 1}$$

By multiplying $(x + 2)^2(x - 1)$, we get

$$9 = A(x + 2)(x - 1) + B(x - 1) + C(x + 2)^2 \quad \text{(i)}$$
$$9 = A(x^2 + x - 2) + B(x - 1) + C(x^2 + 4x + 4) \quad \text{(ii)}$$

As, $x + 2 = 0$

$x = -2$

Put $x = -2$ in (i),

$$9 = A(-2 + 2)(-2 - 1) + B(-2 - 1) + C(-2 + 2)^2$$
$$9 = 0 - 3B + 0$$
$$\Rightarrow \quad 9 = -3B$$
$$\frac{9}{-3} = B$$
$$\Rightarrow \quad \boxed{B = -3}$$

And

$x - 1 = 0$

$x = 1$

Put $x = 1$ in (i),

$$9 = A(1 + 2)(1 - 1) + B(1 - 1) + C(1 + 2)^2$$
$$9 = 0 + 0 + C(3)^2$$
$$9 = 9C$$
$$\frac{9}{9} = 9C$$
$$\Rightarrow \quad \boxed{C = 1}$$

By comparing the coefficients of x^2 from (ii),

$$0 = A + C$$

By putting the value of C,

$$0 = A + 1$$

$$\Rightarrow \quad \boxed{A = -1}$$

By putting the values of A, B, C, we get

$$= \frac{-1}{x+2} - \frac{3}{(x+2)^2} + \frac{1}{x-1}$$

$$= \frac{1}{x-1} - \frac{1}{x+2} - \frac{3}{(x+2)^2}$$

So,

$$\frac{9}{(x-1)(x+2)^2} = \frac{1}{x-1} - \frac{1}{x+2} - \frac{3}{(x+2)^2}$$

Q.7.(a) If A = {1, 2, 3, 4, 5, 6}, B = {2, 4, 6, 8}, C = {1, 4, 8}, then prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. (4)

Ans L.H.S = $A \cap (B \cup C)$

$= A \cap [\{2, 4, 6, 8\} \cup \{1, 4, 8\}]$

$= A \cap [\{1, 2, 4, 6, 8\}]$

$= \{1, 2, 3, 4, 5, 6\} \cap \{1, 2, 4, 6, 8\}$

$= \{1, 2, 4, 6\}$

R.H.S = $(A \cap B) \cup (A \cap C)$

$= [\{1, 2, 3, 4, 5, 6\} \cap \{2, 4, 6, 8\}] \cup [\{1, 2, 3, 4, 5, 6\} \cap \{1, 4, 8\}]$

$= \{2, 4, 6\} \cup \{1, 4\}$

$= \{1, 2, 4, 6\}$

So, L.H.S = R.H.S

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(b) Find the standard deviation 'S' for the set of numbers 12, 6, 7, 3, 15, 10, 18, 5. (4)

Ans

$$\bar{X} = \frac{12 + 6 + 7 + 3 + 15 + 10 + 18 + 5}{8}$$

$$\bar{X} = \frac{76}{8}$$

$\bar{X} = 9.5$

X	$X-\bar{X}$	$(X-\bar{X})^2$
12	2.5	6.25
6	–3.5	12.25
7	–2.5	6.25
3	–6.5	42.25
15	5.5	30.25
10	0.5	0.25
18	8.5	72.25
5	–4.5	20.25
		190

Standard deviation:

$$S.D_{(X)} = S = \sqrt{\frac{\Sigma(X-\bar{X})^2}{n}}$$

$$= \sqrt{\frac{190}{8}}$$

$$= \sqrt{23.75}$$

$$\boxed{S = 4.87}$$

Q.8.(a) Prove that: $\frac{1+\sin\theta}{1-\sin\theta} - \frac{1-\sin\theta}{1+\sin\theta} = 4\tan\theta\sec\theta$. (4

Ans L.H.S $= \frac{1+\sin\theta}{1-\sin\theta} - \frac{1-\sin\theta}{1+\sin\theta}$

$$= \frac{(1+\sin\theta)(1+\sin\theta) - (1-\sin\theta)(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$$

$$= \frac{(1+\sin\theta)^2 - (1-\sin\theta)^2}{(1)^2 - (\sin\theta)^2}$$

$$= \frac{(1+\sin^2\theta + 2\sin\theta) - (1+\sin^2\theta - 2\sin\theta)}{1-\sin^2\theta}$$

$$= \frac{1+\sin^2\theta + 2\sin\theta - 1 - \sin^2\theta + 2\sin\theta}{(\sin^2\theta + \cos^2\theta) - \sin^2\theta}$$

$$= \frac{1 - 1 + \sin^2\theta - \sin^2\theta + 2\sin\theta + 2\sin\theta}{\sin^2\theta + \cos^2\theta - \sin^2\theta}$$

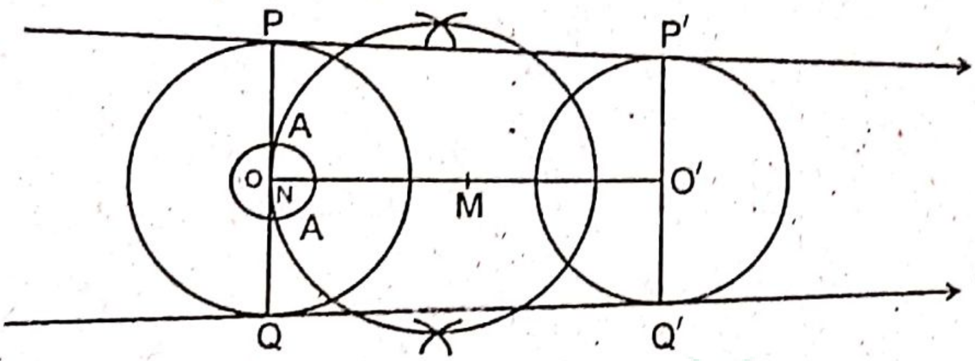
$$= \frac{4\sin\theta}{\cos^2\theta}$$

$$= 4\frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\cos\theta}$$

$$= 4\tan\theta\sec\theta = \text{R.HS} \quad \text{Proved.}$$

(b) Draw two circles with radii 2.5 cm and 3 cm. If their centres are 6.5 cm apart, then draw two direct common tangents. (4)

Ans



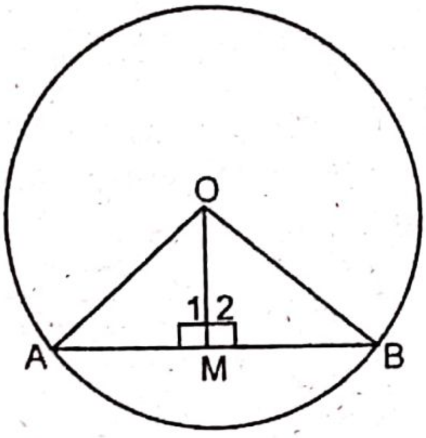
Construction:

1. Draw a line segment $\overline{OO'}$ of length 6.5 cm.
2. Take O as centre and draw a circle with radius 3 cm.
3. Take O′ as centre and draw a circle with radius 2.5 cm.
4. At mid-point of $\overline{OO'}$ called M. Take M as centre point and draw a circle with radius MO′.
5. Cut $m\overline{ON} = 3 - 2.5 = 0.5$ and take O as centre, draw the circle with radius $m\overline{ON}$. This circle intersects the circle C, at point A, A′.
6. Join O with A, A′ and produce on both sides. OA and OA′ produced intersect the larger circle at P and Q.
7. Draw $\overline{O'P'} \parallel \overline{OP}$ and $\overline{O'Q'} \parallel \overline{OQ}$.
8. By joining P with P′ and Q with Q′, we may get the required tangents.

Q.9. Prove that perpendicular from the centre of a circle on a chord bisects it. **(8)**

Ans **Given:**

M is the mid-point of any chord $\overline{AB}$ of a circle which centre at O. Where chord $\overline{AB}$ is not the diameter of the circle.



To prove:

$\overline{OM} \perp$ the chord $\overline{AB}$.

Construction:

Join A and B with centre O. Write $\angle 1$ & $\angle 2$ as shown in the figure.

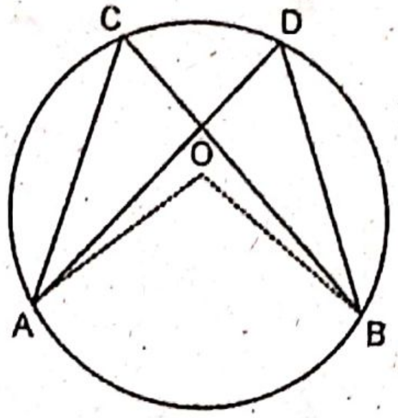
Proof:

Statement	Reasons
In $\Delta OAM \leftrightarrow \Delta OBM$	
$m\overline{OA} = m\overline{OB}$	Radii of same circle
$m\overline{AM} = m\overline{BM}$	Given
$m\overline{OM} = m\overline{OM}$	Common
$\because$ $\Delta OAM \cong \Delta OBM$	S.S.S $\cong$ S.S.S
$\Rightarrow$ $m\angle 1 = m\angle 2$	
i.e., $m\angle 1 + m\angle 2 = m\angle AMB = 180^\circ$	Adjacent supplementary angle
$m\angle 1 = m\angle 2 = 90^\circ$	
$\overline{OM} \perp \overline{AB}$	From (i) & (ii)

OR

Prove that any two angles in the same segment of a circle are equal.

Ans



Given:

m∠ACB = m∠ADB are the circumangles in the same segment of a circle with centre O.

To Prove:

m∠ACB = m∠ADB

Construction:

Join O with A and O with B.

So that ∠AOB is the central angle.

Proof:

Statements	**Reasons**
Standing on the same arc AB of a circle.	
∠AOB is the central angle whereas	Construction
∠ACB and ∠ADB are circumangles	Given
∴ m∠AOB = 2m∠ACB (i)	By theorem I (External angle is the sum internal opposite angle).
and m∠AOB = 2m∠ADB (ii)	By theorem I
⇒ 2m∠ACB = 2m∠ADB	Using (i) and (ii)
Hence, m∠ACB = m∠ADB	